



(e) Is  $f$  odd, even, or neither?

**Solution:**

The graph of  $f$  is neither symmetric about the  $y$ -axis nor the origin so it is *neither* odd or even.

(f) Find  $(f \circ f)(2.5)$ .

**Solution:**

$$(f \circ f)(2.5) = f(f(2.5)) = f(2) = \boxed{0}$$

(g)  $f$  is not one-to-one. Briefly explain why it isn't one-to-one.

**Solution:**

The graph of  $f$  does not pass the horizontal line test. For example,  $f(-2) = 2$  and  $f(2.5) = 2$  but  $-2 \notin 2.5$ .

(h) Identify a restriction of the domain so that  $f$  is one-to-one and has the same range as in part (b).

**Solution:**

$$\boxed{[-2; 2]}$$

(i) Use your domain restriction to calculate  $f^{-1}(2)$ .

**Solution:**

$$f^{-1}(2) = \boxed{-2}$$

(j) Find the  $x$ -values where  $f(x) < 0$ . Give your answer in interval notation.

**Solution:**

$$\boxed{[0; 2)}$$

(k) Find the average rate of change from  $x = -1$  to  $x = 0$ .

**Solution:**

$$\frac{f(-1) - f(0)}{-1 - 0} = \frac{1 - (-2)}{-1} = \boxed{-3}$$

2. The following are unrelated: (9 pts)

(a) Find the equation of the circle centered at  $(-3; 1)$  with radius  $r = \sqrt{5}$ .

**Solution:**

$$(x + 3)^2 + (y - 1)^2 = 5$$

(b) Find the equation of the line that crosses through the points  $(1; -2)$  and  $(4; 3)$ .

**Solution:**

The slope equation gives us  $m = \frac{3 - (-2)}{4 - 1} = \frac{5}{3}$

Using the slope-intercept equation, we get  $y = \frac{5}{3}x + b$ . To solve for  $b$  we substitute in the point  $(4; 3)$ .

$$y = \frac{5}{3}x + b \tag{1}$$

$$3 = \frac{5}{3} \cdot 4 + b \tag{2}$$

$$9 = 20 + 3b \tag{3}$$

$$\frac{11}{3} = b \tag{4}$$

So we get the equation of the line  $y = \frac{5}{3}x + \frac{11}{3}$ .

(c) Find the equation of the vertical line that crosses through the point  $(5; 7)$ .

**Solution:**

$$x = 5$$

3. Suppose  $a$  is a constant and you are given two points  $(-2; a)$  and  $(1; 1)$ . Now suppose you know the midpoint between the two points is  $(\frac{3}{2}; \frac{5}{2})$ . Find the value for  $a$ . (3 pts)

**Solution:**

The  $y$ -coordinate for the midpoint is found by writing  $\frac{a + 1}{2} = \frac{5}{2}$ . Now we can solve for  $a$ :

$$\frac{a + 1}{2} = \frac{5}{2} \tag{5}$$

$$a + 1 = 5 \tag{6}$$

$$a = 4 \tag{7}$$

4. Find the domain of the following functions. Express your answers in interval notation. (15 pts)

$$(a) m(x) = \frac{x-4}{x^2-16}$$

**Solution:**

The domain is all real numbers except when  $x^2 - 16 = 0$ .

$$x^2 - 16 = 0 \quad (8)$$

$$x^2 = 16 \quad (9)$$

$$x = \pm 4 \quad (10)$$

So the domain is  $(-\infty; -4) \cup (-4; 4) \cup (4; \infty)$ .

$$(b) q(r) = \frac{\sqrt[3]{r}}{r-1}$$

**Solution:**

Since the square root of a negative number does not exist in the real numbers, then  $r \geq 0$ . However,  $r - 1$  in the denominator cannot be zero, so  $r \neq 1$ . Thus the domain is  $[0; 1) \cup (1; \infty)$ .

$$(c) r(t) = \sqrt[3]{t-1}$$

**Solution:**

Since the cube root of a negative number results in a negative number, then the domain is  $(-\infty; \infty)$ .

5. For  $f(x) = \sqrt{x-3}$  and  $k(x) = x^2 + 3$ , find the following: (10 pts)

(a) Find  $f(11)$

**Solution:**

$$f(11) = \sqrt{11-3} = \sqrt{8} = 2\sqrt{2}$$

(b) Find  $f(x+12)$

**Solution:**

$$f(x+12) = \sqrt{x+12-3} = \sqrt{x+9}$$

(c) Find  $(k \circ f)(x)$ .

**Solution:**

$$(k \circ f)(x) = k(f(x)) = k(\sqrt{x-3}) = (\sqrt{x-3})^2 + 3 = x - 3 + 3 = x$$

(d) Find the domain of  $(k \circ f)(x)$ .

**Solution:**

The domain of  $(k \circ f)(x)$  is  $[3; 7)$  since  $\sqrt{x-3}$  requires  $x-3 \geq 0$ .

6. Answer the following for the one-to-one function  $g(x) = \frac{1}{x-2}$ . (5 pts)

(a) Find  $g^{-1}(x)$ .

**Solution:**

First we let  $g(x) = y = \frac{1}{x-2}$ . Now we solve for  $x$ :

$$y = \frac{1}{x-2} \tag{11}$$

$$y(x-2) = 1 \tag{12}$$

$$x-2 = \frac{1}{y} \tag{13}$$

$$x = \frac{1}{y} + 2 \tag{14}$$

Swapping  $x$  and  $y$  and substituting the new  $y$  for  $g^{-1}(x)$  we get  $g^{-1}(x) = \frac{1}{x} + 2$ .

(b) What is the range of  $g^{-1}(x)$ ?

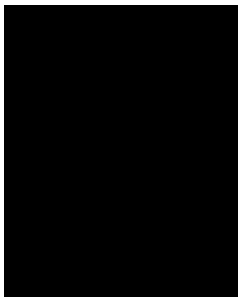
**Solution:**

The range of  $g^{-1}(x)$  is the domain of  $g(x) = \frac{1}{x-2}$  which is  $(-1; 2) \cup (2; 7)$ .

7. Sketch the shape of the graph of each of the following on the provided axes. Make sure to label relevant value(s) on your axis(es) (19 pts)

(a)  $f(x) = x^2$

**Solution:**



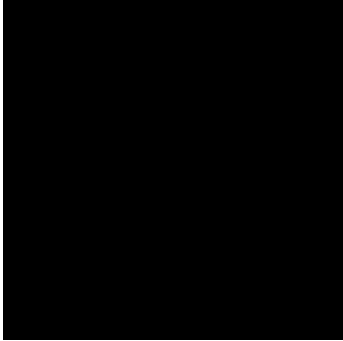
(b)  $k(x) = \sqrt{x-2}$

**Solution:**



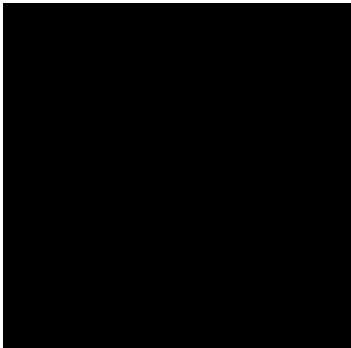
(c)  $(x + 2)^2 + y^2 = 1$

**Solution:**



(d)  $g(x) = \frac{p}{x}$

**Solution:**



(e)  $m(x) = jx + 1j$

**Solution:**



(f)  $q(x) = \begin{cases} 3 & \text{if } x \leq 1 \\ \frac{2}{3}x - 2 & \text{if } x > 1 \end{cases}$

**Solution:** [q2Td821007458796232944cm/m6DoQQon:](#)

(c) Find all zeros and identify the multiplicity of each zero.

**Solution:**

The zeros of a polynomial are the  $x$ -values that result in  $P(x) = 0$ . So we set  $2x^3 + 8x^2 + 8x = 0$ .  
By factoring:

$$2x^3 + 8x^2 + 8x = 0 \quad (15)$$

$$2x(x^2 + 4x + 4) = 0 \quad (16)$$

$$2x(x + 2)(x + 2) = 0 \quad (17)$$

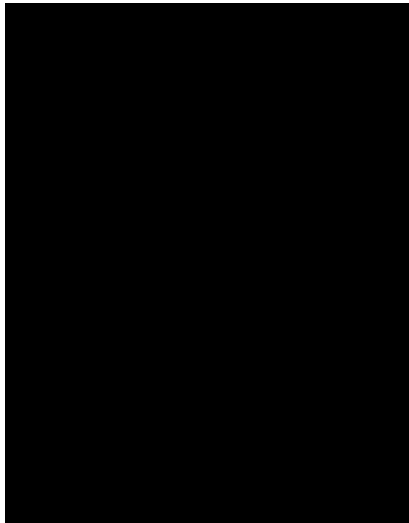
$$2x(x + 2)^2 = 0 \quad (18)$$

So we get  $x = 0$  and  $x = -2$  as the zeros. The multiplicity of  $x = 0$  is  $\boxed{1}$  and  $x = -2$  is  $\boxed{2}$ .

9. Sketch the shape of the graph of a polynomial function,  $g(x)$ , that satisfies **all** of the information. **Label** all intercepts on the graph. (5 pts)

- i. The graph has  $y$ -intercept  $(0; 3)$ .
- ii. The graph has end behavior consistent with  $y = \frac{1}{2}x^4$ .
- iii. The graph crosses at  $(-2; 0)$  and  $(3; 0)$  and bounces (touches but does not cross) at  $(1; 0)$ .
- iv. The graph has no other  $x$ -intercepts.

**Solution:**





10. Use long division to find the quotient and remainder when  $x^3 - 2x^2 + x - 7$  is divided by  $x - 2$ . (4 pts)

**Solution:**

$$\begin{array}{r} x^2 + 1 \\ x - 2 \overline{) x^3 - 2x^2 + x - 7} \\ \underline{(x^3 - 2x^2)} \phantom{+ x - 7} \\ 0 + x - 7 \\ \phantom{0 + } \underline{(x - 2)} \\ \phantom{0 + x - } 5 \end{array}$$

So the quotient is  $x^2 + 1$  and the remainder is  $5$ .

11. The following are unrelated.

(a) Is  $f(x) = x^3 - 2x$  odd, even, or neither? Justify your answer for full credit. (4 pts)

**Solution:**

To determine if  $f(x)$

