(e) Is f odd, even, or neither?

Solution:

The graph of f is neither symmetric about the y-axis nor the origin so it is $\boxed{neither}$ odd or even.

(f) Find $(f \circ f)(2.5)$.

Solution:

$(f \circ f)(2.5) = f(f(2.5)) = f(2) = \boxed{0}$

(g) f is not one-to-one. Briefly explain why it isn't one-to-one.

Solution:

$\boxed{\text{The graph of } f \text{ does not pass the horizontal line test}}$. For example, $f(-2) = 2$ and $f(2.5) = 2$ but $-2 \neq 2.5$.

(h) Identify a restriction of the domain so that f is one-to-one and has the same range as in part (b).

Solution:

$\boxed{[-2, 2]}$

(i) Use your domain restriction to calculate $f^{-1}(2)$.

Solution:

$f^{-1}(2) = \boxed{-2}$

(j) Find the x-values where $f(x) < 0$. Give your answer in interval notation.

Solution:

$\boxed{[0, 2)}$

(k) Find the average rate of change from $x = -1$ to $x = 0$.

Solution:

$$\frac{f(-1) - f(0)}{-1 - 0} = \frac{1 - (-2)}{-1} = \boxed{-3}$$

2. The following are unrelated: (9 pts)

(a) Find the equation of the circle centered at $(-3, 1)$ with radius $r = \sqrt{5}$.

Solution:

$$\boxed{(x+3)^2 + (y-1)^2 = 5}$$

(b) Find the equation of the line that crosses through the points $(1, -2)$ and $(4, 3)$.

Solution:

The slope equation gives us $m = \dfrac{3-(-2)}{4-1} = \dfrac{5}{3}$

Using the slope-intercept equation, we get $y = \frac{5}{3}x + b$. To solve for b we substitute in the point $(4, 3)$.

$$y = \frac{5}{3}x + b \quad (1)$$

$$3 = \frac{5}{3}4 + b \quad (2)$$

$$9 = 20 + 3b \quad (3)$$

$$-\frac{11}{3} = b \quad (4)$$

So we get the equation of the line $\boxed{y = \frac{5}{3}x - \frac{11}{3}}$.

(c) Find the equation of the vertical line that crosses through the point $(5, 7)$.

Solution:

$$\boxed{x = 5}$$

3. Suppose a is a constant and you are given two points $(-2, a)$ and $(-1, 1)$. Now suppose you know the midpoint between the two points is $\left(-\frac{3}{2}, \frac{5}{2}\right)$. Find the value for a. (3 pts)

Solution:

The y-coordinate for the midpoint is found by writing $\dfrac{a+1}{2} = \dfrac{5}{2}$. Now we can solve for a:

$$\frac{a+1}{2} = \frac{5}{2} \quad (5)$$

$$a + 1 = 5 \quad (6)$$

$$a = \boxed{4} \quad (7)$$

4. Find the domain of the following functions. Express your answers in interval notation. (15 pts)

(a) $m(x) = \dfrac{x - 4}{x^2 - 16}$

Solution:

The domain is all real numbers except when $x^2 - 16 = 0$.

$$x^2 - 16 = 0 \tag{8}$$
$$x^2 = 16 \tag{9}$$
$$x = \pm 4 \tag{10}$$

So the domain is $\boxed{(-\infty; -4) \cup (-4; 4) \cup (4; \infty)}$.

(b) $q(r) = \dfrac{3\sqrt{r}}{r - 1}$

Solution:

Since the square root of a negative number does not exist in the real numbers, then $r \geq 0$. However, $r - 1$ in the denominator cannot be zero, so $r \neq 1$. Thus the domain is $\boxed{[0; 1) \cup (1; \infty)}$.

(c) $r(t) = 5\sqrt[3]{t - 1}$

Solution:

Since the cube root of a negative number results in a negative number, then the domain is $\boxed{(-\infty; \infty)}$.

5. For $f(x) = \sqrt{x - 3}$ and $k(x) = x^2 + 3$, find the following: (10 pts)

(a) Find $f(11)$

Solution:

$f(11) = \sqrt{11 - 3} = \sqrt{8} = \boxed{2\sqrt{2}}$

(b) Find $f(x + 12)$

Solution:

$f(x + 12) = \sqrt{x + 12 - 3} = \boxed{\sqrt{x + 9}}$

(c) Find $(k \circ f)(x)$.

Solution:

$(k \circ f)(x) = k(f(x)) = k\left(\sqrt{x - 3}\right) = \left(\sqrt{x - 3}\right)^2 + 3 = x - 3 + 3 = \boxed{x}$.

(d) Find the domain of $(k \circ f)(x)$.

Solution:

The domain of $(k \circ f)(x)$ is $[3, \infty)$ since $\sqrt{x - 3}$ requires $x - 3 \geq 0$.

6. Answer the following for the one-to-one function $g(x) = \frac{1}{x - 2}$. (5 pts)

(a) Find $g^{-1}(x)$.

Solution:

First we let $g(x) = y = \frac{1}{x - 2}$. Now we solve for x:

$$y = \frac{1}{x - 2} \quad (11)$$

$$y(x - 2) = 1 \quad (12)$$

$$x - 2 = \frac{1}{y} \quad (13)$$

$$x = \frac{1}{y} + 2 \quad (14)$$

Swapping x and y and substituting the new y for $g^{-1}(x)$ we get $\boxed{g^{-1}(x) = \frac{1}{x} + 2}$.

(b) What is the range of $g^{-1}(x)$?

Solution:

The range of $g^{-1}(x)$ is the domain of $g(x) = \frac{1}{x - 2}$ which is $\boxed{(-\infty, 2) \cup (2, \infty)}$.

7. Sketch the shape of the graph of each of the following on the provided axes. Make sure to label relevant value(s) on your axe(s) (19 pts)

(a) $f(x) = -x^2$

Solution:

[redacted]

(b) $k(x) = \sqrt[3]{x} - 2$

Solution:

[redacted]

(c) $(x+2)^2 + y^2 = 1$

Solution:

[redacted]

(d) $g(x) = \sqrt{x}$

Solution:

[redacted]

(e) $m(x) = jx + 1j$

Solution:

[redacted]

(f) $q(x) = \begin{cases} 3 & \text{if } x \quad 1 \\ \frac{2}{3}x \quad 2 & \text{if } x > 1 \end{cases}$

Solution:q2Td821007158796232944cmIm6DoQQon:

(c) Find all zeros and identify the multiplicity of each zero.

Solution:

The zeros of a polynomial are the x-values that result in $P(x) = 0$. So we set $2x^3 + 8x^2 + 8x = 0$. By factoring:

$$2x^3 + 8x^2 + 8x = 0 \tag{15}$$

$$2x \;\; x^2 + 4x + 4 \;\; = 0 \tag{16}$$

$$2x(x + 2)(x + 2) = 0 \tag{17}$$

$$2x(x + 2)^2 = 0 \tag{18}$$

So we get $x = 0$ and $x = \quad 2$ as the zeros. The multiplicity of $x = 0$ is $\boxed{1}$ and $x = \quad 2$ is $\boxed{2}$.

9. Sketch the shape of the graph of a polynomial function, $g(x)$, that satisfies **all** of the information. **Label** all intercepts on the graph. (5 pts)

    i. The graph has y-intercept $(0; 3)$.

    ii. The graph has end behavior consistent with $y = \frac{1}{2}x^4$.

    iii. The graph crosses at $(\quad 2; 0)$ and $(3; 0)$ and bounces (touches but does not cross) at $(1; 0)$.

    iv. The graph has no other x-intercepts.

Solution:

[redacted]

10. Use long division to find the quotient and remainder when $x^3 \quad 2x^2 + x \quad 7$ is divided by $x \quad 2$. (4 pts)

Solution:

$$
\begin{array}{r|rrrr}
 & x^2 + & 1 & & \\
\hline
x \quad 2 & x^3 & 2x^2 + & x & 7 \\
 & (x^3 & 2x^2) & & \\
\hline
 & & 0 + & x & 7 \\
 & & & (x & 2) \\
\hline
 & & & & 5
\end{array}
$$

So the quotient is $x^2 + 1$ and the remainder is $\quad 5$.

11. The following are unrelated.

(a) Is $f(x) = x^3 \quad 2x$ odd, even, or neither? Justify your answer for full credit. (4 pts)

Solution:

To determine if $f(x$