

APPM 2350-Exam 3
 Wednesday April 13th, 6:30pm-8pm 2022

This exam has 4 problems. Please start each new problem at the top of a new page in your blue book. Show all your work in your blue book and simplify your answers. Answers with missing or insufficient justification will receive no points. You are allowed one 8.5x11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Problem 1(20 points)

Suppose the temperature at any point in the plane is given by

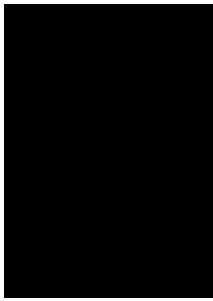
$$T(x; y) = xe^{-y^2}$$

Suppose the area of a region R is given by

$$\int_0^3 \int_{x^2}^9 dy dx$$

- (a) Sketch and shade the region R and clearly label your axes and any intercepts.
- (b) Find the average temperature on the region R . Fully simplify your final answer.

SOLUTION :



(a)

(b) The average temperature is given by:

$$\bar{T} = \frac{\int_R T(x; y) dA}{\int_R dA} = \frac{\int_0^3 \int_{x^2}^9 xe^{-y^2} dy dx}{\int_0^3 \int_{x^2}^9 dy dx}$$

Considering each integral separately and starting with the denominator:

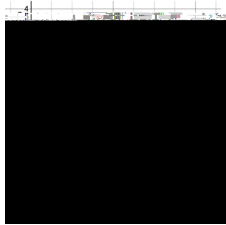
$$\int_0^3 \int_{x^2}^9 dy dx$$

Problem 2(30 points)

The volume of an object is given by

$$\int_0^1 \int_0^{1-\theta} \int_0^{1-\theta-\phi} r \, dz \, dr \, d\theta + \int_0^1 \int_0^{\theta} \int_0^{1-\theta-\phi} r \, dz \, dr \, d\theta$$

(a) Sketch and shade a 2D cross section of the object in the $\theta\phi$ -plane for any such that $\theta =$ _____ = _____



The arrow enters the region when $\rho = 3 \Rightarrow \cos = \frac{\rho}{3} \Rightarrow \rho = 3 \sec$
 The arrow exits the region when $\rho = 3 \Rightarrow \cos = \frac{\rho}{3} \Rightarrow \rho = 3 \sec$
 To find the limits, we note that the smallest ρ occurs when $\theta = 0$.
 The largest occurs when $\theta = \frac{\pi}{3} \Rightarrow \cos = \frac{\rho}{3} \sin$
 $\Rightarrow \tan = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$
 The limits are the same as in cylindrical: $\theta = 0$ to $\theta = \frac{\pi}{6}$
 We replaced $dz dr d\theta$ with $\rho^2 \sin \theta d\rho d\theta d\phi$

$$\Rightarrow 5 = \iint_R \frac{|\mathbf{j} \cdot \mathbf{r}_g|}{|\mathbf{j} \cdot \mathbf{r}_g|} dA$$

Since we are projecting onto the plane $\mathbf{n} = \hat{\mathbf{j}}$

Let $g(x; y; z) = 3x - 4y + 2z$

$$\Rightarrow \mathbf{r}_g = \langle 3, -4, 2 \rangle \Rightarrow |\mathbf{j} \cdot \mathbf{r}_g| = \sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{29}$$

and $|\mathbf{j} \cdot \hat{\mathbf{j}}| = 1$

Thus

$$5 = \iint_R \frac{\sqrt{29}}{\sqrt{29}} dA$$

$$\Rightarrow 5 = \iint_R 1 dA$$

$$\Rightarrow \text{Area of } R = \iint_R dA = \boxed{\frac{15}{29}}$$

Problem 4 (20 points)

Consider the region R , that is bounded by

$$y = \frac{1}{x}; \quad y = \frac{1}{x} + 2; \quad y = 4 - \frac{1}{x}; \quad y = 6 - \frac{1}{x}$$

where x and y are measured in meters.

A sprinkler sprays water on the region such a way that the depth of water (in meters) that reaches the point $(x; y)$ in 1 hour is given by

$$g(x; y) = \frac{e^{(y - \frac{1}{x})}}{20x}$$

Use an appropriate u -transformation to find the total volume of water the sprinkler sprays on the region R in 1 hr. Fully simplify your final answer.

SOLUTION :

$$\text{Total Volume} = \iint_R g(x; y) dA = \iint_R \frac{e^{(y - \frac{1}{x})}}{20x} dA$$

The original region looks like:

Solving for the x -transformation yields:

$$x = \frac{v - u}{2}$$

$$y = \frac{u + v}{2}$$

Thus our boundaries convert as follows:

Boundary in xy	Corresponding Boundary in uv
$y = \frac{v}{x}$	$u = 0$
$y = \frac{v}{x} + 2$	$u = 2$
$y = 4 - \frac{v}{x}$	$v = 4$
$y = 6 - \frac{v}{x}$	$v = 6$

